On the role of message-passing in networked systems

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Large complex networked systems are omnipresent in the modern world. Canonical examples include: (1) communication networks, such as the Internet; (2) statistical networks, such as Markov Random Fields (MRF); or (3) real-world networks such as road transportation networks or social networks (e.g. Facebook). Algorithms or mechanisms exist, in some form or the other, in all complex networked systems as fundamental operational primitives. For an engineered system like the Internet, algorithms are in-built by design; for a real-world network like a road transportation network, mechanisms modulate the behavior of agents or humans based on a model of their essential characteristics. A key intellectual commonality in understanding these disparate systems is that they are typically governed by a collection of simple, local (algorithmic) rules designed to achieve some specific global, network-wide objective. This is because, in a large engineered system, the desirability of a scalable and easily realizable architecture leads inevitably to the design of local, simple algorithms. For example, consider an algorithm operating in the hardware of a high-aggregate bandwidth Internet router. Usually, it has to make complex decisions in few nanoseconds with its logic spread across chips in the hardware and only so much can be communicated across chips at such a fast time scale. Similarly, in a wireless network, transmitters need to make transmission decision while avoiding interference with little or no global information. Finally, in a real-world network, humans or agents naturally act according to simple rules using local information. Therefore, designing and understanding such simple, local and iterative algorithms is essential. Important class of algorithms that fall in this category are the so-called message-passing algorithms.

Roughly speaking, there are two classes of message-passing algorithms:

1. Fixed-point based algorithms: these algorithms attempt, in an iterative manner, to emulate some specified local fixed-point characterization of a desired global network state. A classical example of such an algorithm is the Metropolis-Hastings' rule that designs the transition matrix of a reversible Markov chain with desired global stationary distribution through the local detailed balance condition. In the context of optimization, the approximate dynamic programming method that attempts to emulate the HJB equation is another such example. Belief propagation (and its variants) is another such fixed point based algorithm which is an approximation based on the dynamic programming fixed point for tree structured graphs.

2. Potential function based algorithms: these algorithms are based on making local updates to the state so as to improve certain potential function; and a state with extreme potential function value corresponds to a desired global network state. The classical gradient based algorithms from optimization are primary examples. A large suite of local, greedy algorithms fall squarely under this umbrella. The recently popular sub-gradient algorithm or now the classical simplex should be viewed as potential function based algorithms. This includes modeling or mechanisms inspired by rational myopic behavior of agents in game theoretic setup.

The important intellectual challenges are:

1. Understanding strengths and limitations of these two broad classes of message-passing algorithms. In particular, studying the effectiveness of such algorithms for various types of prevalent structures is an important quest. For example, belief propagation inspired algorithms for optimization problems seem to

have connections with a certain class of linear programming solvable structures. Such an understanding can guide the design of engineered system as well as help verify appropriateness of certain message-passing mechanism as a behavioral model for a real-world network.

2. The state of a functioning networked system continually changes over time. However, understanding the effect of dynamics as well as designing algorithms to adapt to the dynamics in general has been an outstanding challenge. For example, a quantification of some sorts of the inherent "uncertainty principles" in achieving a specified level of algorithmic performance under a specified degree of dynamics is operationally of great relevance.

In summary, message-passing algorithms have become a promising prototype for algorithms for existing and emerging networked systems. There has been a long history on this topic, and their growing relevance have brought them to the intellectual forefront across disciplines including algorithms, communication, control, data mining, machine learning, networks, signal processing and statistical inference. And undoubtedly they will provide the foundations for the theory and practice of future networked systems.