Introduction to the Control and Optimization Panel Discussion

Keith Glover
Schedule

• Keith Glover – Introduction
• Pablo Parillo – Games and Distributed Decisions
• Stephen Boyd – Optimization and Decisions
• Albert Benveniste – Componentizing and Distributing Feedback
• Jonathan How – Grand Challenges
• Richard Murray – Control of Complex Systems
• Vincent Blondel – Optimization (What’s hard?)
Essence of Control?

- Feedback
- Uncertainty
- The notion of State
- Approximation
- Verification
Approximation

- Modelling the phenomena
  - e.g. HCCI combustion model
    - 2 species + 2 reactions
    - 157 species + 1552 reactions
Approximation

• Modelling the phenomena
• Approximation of Math model with a simpler one

  e.g. Model Reduction
  - H-infty norm
  - approx. with Hankel norm
  - get bounds
  - balanced truncation
  - frequency weighted??
Approximation

- Modelling the phenomena
- Approximation of Math model with a simpler one
- Approximation of objectives

  e.g. choice of norm, weights etc. Typically choose one objective that addresses the most important features and then ‘patch it up’ to address other criteria. e.g.

  - H-infty norm addressing dynamics/robustness with anti-windup for saturation.
  - MPC for input saturation with robustness add-on/analysis.
  - Adaptive control with jacketing software.
Approximation

• Modelling the phenomena
• Approximation of Math model with a simpler one
• Approximation of objectives
• Approximate optimisation

e.g.
- just use small number of iterations in real-time optimisation. (Boyd)
- Sum of Squares bounds (Parillo)
Approximation

- Modelling the phenomena
- Approximation of Math model with a simpler one
- Approximation of objectives
- Approximate optimisation
- Bounds on resulting behaviour.

All the available analysis tools from IQC’s, LMI’s, hybrid systems … preferably as part of the design but also post-facto for any ad hoc design.
Verification/Certification

- Bounds on behaviour as before.
- Finite state elements
- Code validation (CS).
- Failure detection, reconfig.
- In aerospace and automotive, certification is perhaps the biggest obstacle to real-time optimisation.
Legitimate Academic Pursuits

- Deeper understanding of (feedback) systems.
- e.g. limitations in general (Bode)
- Specific behaviour (climate change, human biology)
Summary: Limitations due to channel capacity

System:

\[ x_1 \xrightarrow{u_t} x_1 + 1 \]
\[ x_1 \xrightarrow{x_1/\tau_1} x_1 - 1 \]

\[ dx_1 = u_t dt + \sqrt{2\langle x_1 \rangle / \tau_1} dw \]

Sensor:

\[ x_2 \xrightarrow{\alpha x_1} x_2 + 1 \]
\[ x_2 \xrightarrow{x_2/\tau_2} x_2 - 1 \]

where \( u_t = f(\{x_2(t') : t' < t\}) \)

\[ \frac{\sigma_1^2}{\langle x_1 \rangle} \geq \frac{1}{\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{N_2}{N_1}}} \]

\[ \approx \begin{cases} 
1, & N_2 < N_1 \\
\sqrt{\frac{N_1}{N_2}}, & N_2 > N_1 
\end{cases} \]

where \( N_2 = \langle x_2 \rangle \tau_1 / \tau_2 = \) no of molecules of \( X_2 \) made per lifetime of \( X_1 \).
\( N_1 = \langle x_1 \rangle = \) no of molecules of \( X_1 \) made per lifetime of \( X_1 \).
Legitimate Academic Pursuits?

- Deeper understanding of (feedback) systems.
- Design methodologies for certain application areas.
- Robust control paradigm
- MPC
Legitimate Academic Pursuits?

- Deeper understanding of (feedback) systems.
- Design methodologies for certain application areas.
- Algorithmic advances and limitations.

- Bounds
- NP hard
- Speed/efficiency
## Stability Analysis Using Sum of Squares

### Nonlinear system:

\[ \dot{x} = f(x), \quad x \in \mathbb{R}^n, \quad x(0) = X_0 \]

Construct \( V(x), \varphi_1(x) > 0, \varphi_2(x) > 0 \) s.t.

- \( V(x) - \varphi_1(x) \) is SOS
- \( -\frac{dV}{dx} f(x) - \varphi_2(x) \) is SOS

Then use SOSTOOLS.

### Problem:

The size of the underlying SDP grows rapidly as \( n \) increases. Currently \( n > 8 \) is difficult without taking into account system structure (e.g., sparsity, symmetry).

### Example: Ecological network with community matrix \( A \) and species birth rate \( b \):

\[ \dot{x}_i = x_i \left( b_i - x_i - \sum_{j=1}^{16} A_{ij} x_j \right), \quad x \in \mathbb{R}^{16} \]

Decompose system so as to minimize energy flow between subsystems:

Smaller SDP is solved for composite system than for the complete system.

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Legitimate Academic Pursuits

• Deeper understanding of (feedback) systems.
• Design methodologies for certain application areas.
• Algorithmic advances and limitations.
• Verification tools.
Legitimate Academic Pursuits?

- Deeper understanding of (feedback) systems.
- Design methodologies for certain application areas.
- Algorithmic advances and limitations.
- Verification tools.
- Case Studies

- Demonstrators of power/applicability of methods – flight control
- Demonstrators of potential technological solutions – DARPA grand challenges.
- Solving specific problems for a practitioner.
- Identifying more generic open problems – hybrid systems.
\begin{align*}
\dot{x} &= f(x, \theta), \quad x \in X \subseteq \mathbb{R}^{n_x}, \\
\theta &\in \Theta \subseteq \mathbb{R}^{n_{\theta}}, \quad x(0) \in X_0, \\
\text{Unsafe states: } x &\in X_u
\end{align*}

Find \( B(x) \) satisfying:
\begin{align*}
B(x) &< 0 \text{ in } X_0 \\
B(x) &\geq 0 \text{ in } X_u \\
\frac{\partial B}{\partial x} f(x, \theta) &\leq 0 \text{ for } x \in X, \\
\theta &\in \Theta
\end{align*}

- 10-dimensional nonlinear model, with 6 modes.
- A function \( B(x) \) was constructed, which guarantees controller will not result in unsafe operation for all uncertainty combinations.

IEEE TCT, Nov 07

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Malcolm Smith – the inerter

Williams FW14B driven by Nigel Mansell in 1992
The first championship winning car to use active suspension

The Inerter — Origin of the Idea

- Applied Work on Active Suspension (Formula One)
- Theory Work on Active Suspension
- Theory Work on Passive Suspension

... a curious lack of symmetry in basic modelling ...
## A new correspondence for network synthesis

<table>
<thead>
<tr>
<th>Mechanical</th>
<th>Electrical</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{v_2}$</td>
<td>$i_{v_1}$</td>
</tr>
<tr>
<td>$\frac{dF}{dt} = k(v_2 - v_1)$</td>
<td>$\frac{di}{dt} = \frac{1}{L}(v_2 - v_1)$</td>
</tr>
<tr>
<td>$F_{v_2}$</td>
<td>$i_{v_1}$</td>
</tr>
<tr>
<td>$F = b \frac{d(v_2 - v_1)}{dt}$</td>
<td></td>
</tr>
<tr>
<td>$F = c(v_2 - v_1)$</td>
<td>$i = \frac{1}{R}(v_2 - v_1)$</td>
</tr>
</tbody>
</table>

Y(s) = admittance = \frac{1}{impedance}

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**First Formula One Grand Prix for the inverter**

Raced by Kimi Raikkonen at the 2005 Spanish Grand Prix in Barcelona.

Raikkonen won the race to give McLaren their first victory of the 2005 season.
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