Introduction to the Control and Optimization Panel Discussion

Keith Glover

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Department of Engineering

Schedule

- Keith Glover – Introduction
- Pablo Parillo – Games and Distributed Decisions
- Stephen Boyd – Optimization and Decisions
- Albert Benveniste – Componentizing and Distributing Feedback
- Jonathan How – Grand Challenges
- Richard Murray – Control of Complex Systems
- Vincent Blondel – Optimization (What's hard?)

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Essence of Control?

- Feedback
- Uncertainty
- The notion of State
- Approximation
- Verification

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Approximation

- Modelling the phenomena

e.g. HCCI combustion model

– 2 species + 2 reactions

–157 species +1552 reactions

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Approximation

- Modelling the phenomena
- Approximation of Math model with a simpler one

e.g. Model Reduction

- H-infty norm
- approx. with Hankel norm
- get bounds
- balanced truncation
- frequency weighted??

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Approximation

- Modelling the phenomena
- Approximation of Math model with a simpler one
- Approximation of objectives

e.g. choice of norm, weights etc.

Typically choose one objective that addresses the most important features and then 'patch it up' to address other criteria. e.g.

- H-infty norm addressing dynamics/robustness with anti-windup for saturation.
- MPC for input saturation with robustness add-on/analysis.
- Adaptive control with jacketing software.

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Approximation

- Modelling the phenomena
- Approximation of Math model with a simpler one
- Approximation of objectives
- Approximate optimisation

e.g.

- just use small number of iterations in real-time optimisation. (Boyd)
- Sum of Squares bounds (Parillo)

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Approximation

- Modelling the phenomena
- Approximation of Math model with a simpler one
- Approximation of objectives
- Approximate optimisation
- Bounds on resulting behaviour.

All the available analysis tools from IQC's, LMI's, hybrid systems …

preferably as part of the design but also post-facto for any ad hoc design.

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Verification/Certification

- Bounds on behaviour as before.
- Finite state elements
- Code validation (CS).
- Failure detection, reconfig.
- In aerospace and automotive, certification is perhaps the biggest obstacle to real-time optimisation.

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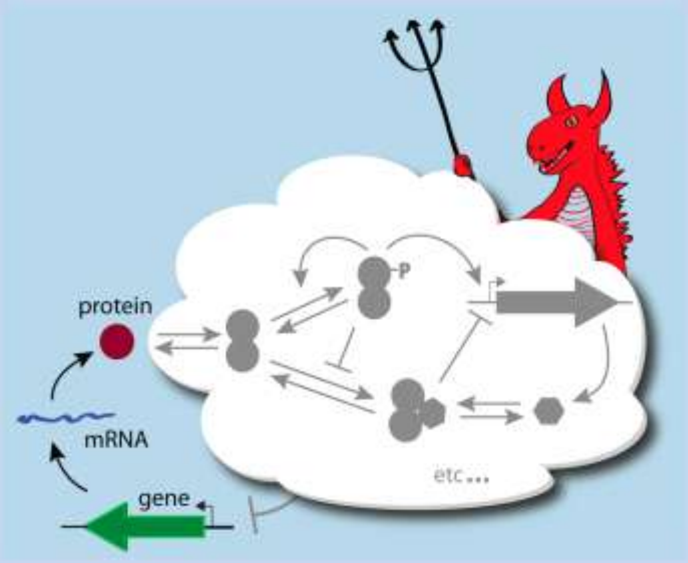
Legitimate Academic Pursuits ?

- Deeper understanding of (feedback) systems.
- e.g. limitations in general (Bode)
- Specific behaviour (climate change, human biology)

Fundamental limits on the suppression of molecular fluctuations



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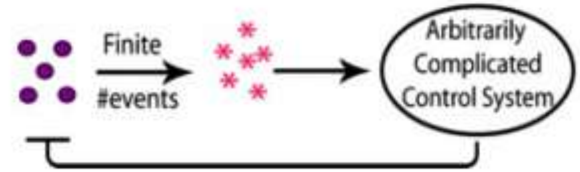
Vinnicombe, Lestas and Paulsson

Summary: Limitations due to channel capacity



System: $\begin{array}{l} x_1 \xrightarrow{u_t} x_1 + 1 \\ x_1 \xrightarrow{x_1/\tau_1} x_1 - 1 \end{array}$ ⇒ $dx_1 = u_t dt + \sqrt{2\langle x_1 \rangle / \tau_1}\, dw$

Sensor: $\begin{array}{l} x_2 \xrightarrow{\alpha x_1} x_2 + 1 \\ x_2 \xrightarrow{x_2/\tau_2} x_2 - 1 \end{array}$

where $u_t = f(\{x_2(t') : t' < t\})$

$$\frac{\sigma_1^2}{\langle x_1 \rangle} \geq \frac{1}{\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{N_2}{N_1}}} \approx \begin{cases} 1, & N_2 < N_1 \\ \sqrt{\frac{N_1}{N_2}}, & N_2 > N_1 \end{cases}$$

where $N_2 = \langle x_2 \rangle \tau_1 / \tau_2$ = no of molecules of X_2 made per lifetime of X_1.
$N_1 = \langle x_1 \rangle$ = no of molecules of X_1 made per lifetime of X_1.

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Legitimate Academic Pursuits ?

- Deeper understanding of (feedback) systems.
- Design methodologies for certain application areas.

- Robust control paradigm
- MPC

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Legitimate Academic Pursuits ?

- Deeper understanding of (feedback) systems.
- Design methodologies for certain application areas.
- Algorithmic advances and limitations.

- Bounds
- NP hard
- Speed/efficiency

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Antonis Papachristodoulou

Stability Analysis Using Sum of Squares

Nonlinear system:

$$\dot{x} = f(x), \quad x \in \mathbb{R}^n \quad , x(0) = X_0$$

Construct $V(x)$, $\varphi_1(x) > 0$, $\varphi_2(x) > 0$ s.t.

$V(x) - \varphi_1(x)$ is SOS

$-\frac{dV}{dx} f(x) - \varphi_2(x)$ is SOS

Then use SOSTOOLS.

Problem:

The size of the underlying SDP grows rapidly as n increases. Currently $n > 8$ is difficult without taking into account system structure (e.g., sparsity, symmetry).

Approach:

1) Automatically decompose $\dot{x} = f(x)$ into:

$$\dot{x}_1 = f_1(x_1) + g_1(x_1, x_2), \; x_1 \in \mathbb{R}^{n_1}$$
$$\dot{x}_2 = f_2(x_2) + g_2(x_1, x_2), \; x_2 \in \mathbb{R}^{n_2}$$

where $n_1 + n_2 = n$

2) Construct two Lyapunov functions, $V_i(x_i)$ for $\dot{x}_i = f_i(x_i)$.

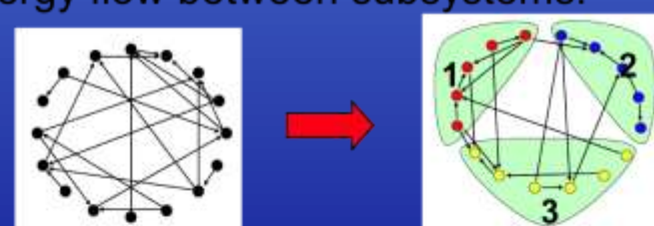
3) Search for $\alpha > 0$ such that $V(x) = V_1(x_1) + \alpha V_2(x_2)$ is a Lyapunov function for the original system.

Method extends to multiple partitions and allows the analysis of larger systems.

Example: Ecological network with community matrix A and species birth rate b:

$$\dot{x}_i = x_i \left(b_i - x_i - \sum_{i=1}^{j} A_{ij} x_j \right), \; x \in \mathbb{R}^{16}$$

Decompose system so as to minimize energy flow between subsystems:



Smaller SDP is solved for composite system than for the complete system.

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Legitimate Academic Pursuits ?

- Deeper understanding of (feedback) systems.
- Design methodologies for certain application areas.
- Algorithmic advances and limitations.
- Verification tools.

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Legitimate Academic Pursuits ?

- Deeper understanding of (feedback) systems.
- Design methodologies for certain application areas.
- Algorithmic advances and limitations.
- Verification tools.
- Case Studies

- Demonstrators of power/applicability of methods – flight control
- Demonstrators of potential technological solutions – DARPA grand challenges.
- Solving specific problems for a practitioner.
- Identifying more generic open problems – hybrid systems.

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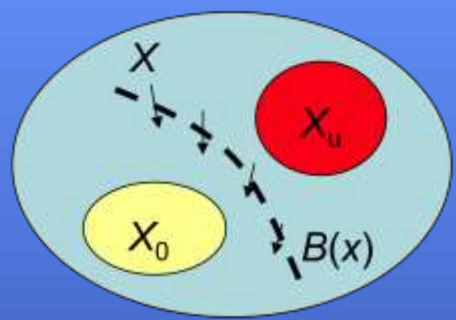
Safety Analysis using Sum of Squares: Life Support System

$\dot{x} = f(x,\theta), \quad x \in X \subseteq \mathbb{R}^{n_x},$

$\theta \in \Theta \subseteq \mathbb{R}^{n_p}, \; x(0) \in X_0,$

Unsafe states: $x \in X_u$



Find $B(x)$ satisfying:

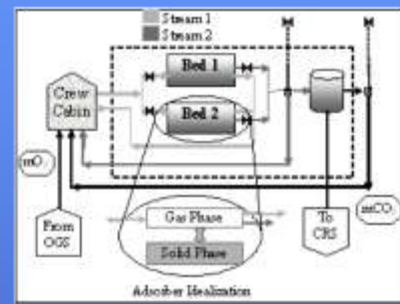
$B(x) < 0$ in X_0

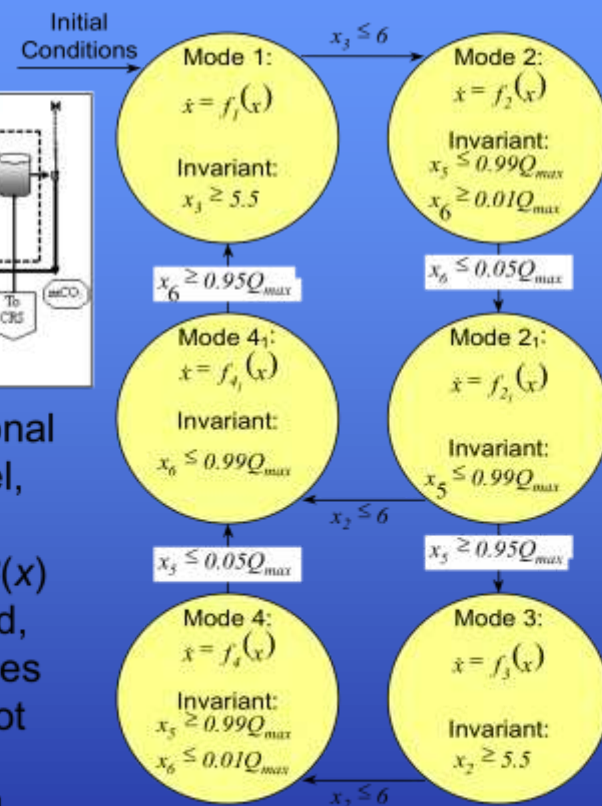
$B(x) \geq 0$ in X_u

$\frac{\partial B}{\partial x} f(x,\theta) \leq 0$ for $x \in X,$

$\theta \in \Theta$

Example: Life Support System





- 10-dimensional nonlinear model, with 6 modes.
- A function $B(x)$ was constructed, which guarantees controller will not result in unsafe operation for all uncertainty combinations

IEEE TCT, Nov 07

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Malcolm Smith – the inerter

Williams FW14B driven by Nigel Mansell in 1992

The first championship winning car to use active suspensio

The Inerter — Origin of the Idea

Applied Work on Active Suspension (Formula One)

↓

Theory Work on Active Suspension

↓

Theory Work on Passive Suspension

↓

... a curious lack of symmetry in basic modelling ...

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Mechanical	Electrical
$Y(s) = \frac{k}{s}$ $\frac{dF}{dt} = k(v_2 - v_1)$ spring	$Y(s) = \frac{1}{Ls}$ $\frac{di}{dt} = \frac{1}{L}(v_2 - v_1)$ inductor
$Y(s) = bs$ $F = b\frac{d(v_2 - v_1)}{dt}$ inerter	$Y(s) = Cs$ $i = C\frac{d(v_2 - v_1)}{dt}$ capacitor
$Y(s) = c$ $F = c(v_2 - v_1)$ damper	$Y(s) = \frac{1}{R}$ $i = \frac{1}{R}(v_2 - v_1)$ resistor

$$Y(s) = \text{admittance} = \frac{1}{\text{impedance}}$$

First Formula One Grand Prix for the inerter

Raced by Kimi Raikonnen at the 2005 Spanish Grand Prix in Barcelona.

Raikonnen won the race to give McLaren their first victory of the 2005 season.

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