What to do about Hard Problems That Don’t Go Away

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Thanks to Anant Sahai for discussions.
Holy Grail of Network Information Theory

What is the optimal achievable performance?
Point-to-Point Communication

\[ C = \max_{p_X} I(X;Y) \quad R(D) = \min_{p_{\hat{X}|X}, E[\rho(\hat{X},X)] \leq D} I(\hat{X};X) \]

Target distortion D is achievable iff \( R(D) < C \).

This result is remarkable but also sets a high bar for us.
Linear Quadratic Gaussian Networks

point-to-point (Shannon 48)
\[ C = \log_2(1 + \text{SNR}) \]

multiple-access (Alshwede, Liao 70’s)
broadcast (Cover, Bergmans, Gallager 70’s)
What We Don’t Know

Unfortunately we don’t know the capacity of most other Gaussian networks.

Interference

(Best known achievable region: Han & Kobayashi 81)

(Best known achievable region: El Gamal & Cover 79)
30 Years Have Gone by.....

We are still stuck.

How to make progress?

Approximate.

But how to approximate?
An Abstraction of an Abstraction

Transmit a real number

\[ x = 0.b_1 b_2 b_3 b_4 b_5 \]

Gaussian channel

Least significant bits are truncated at noise level.
Deterministic Bridge

Gaussian

Deterministic

Wireline

Approximate max-flow min-cut

Generalized max-flow min-cut

Classical max-flow min-cut

\[ C = \min_{\Omega} \text{cut}(\Omega; \Omega^c) \]
Back to Canonical Problems

Interference
Capacity to within 1 bit (Etkin, T. & Wang 06)

Capacity to within 1 bit (Avestimehr, Diggavi & T. 08)
From Information to Control

(Ho, Kastner & Wong 78)
Witsenhausen Counterexample Revisited

\[ \min k u_1^2 + x_2^2 \]

control

\[ x_0 \sim \mathcal{N}(0, \sigma_0^2) \]

\[ x_1 \]

\[ \begin{array}{c}
\mathcal{C}_1 \\
\end{array} \]

\[ u_1 \]

\[ \mathcal{C}_2 \]

\[ w \sim \mathcal{N}(0, 1) \]

estimate

\[ b_1 b_2 \cdots b_n b_{n+1} \cdots \]

Constant-factor optimality.
(Park, Grover & Sahai 09)
Lessons Learnt

• Don’t be obsessed with a specific model.

• Don’t be obsessed with exact solutions.

• Be obsessed with basic phenomena in one’s field.

• Making progress on core problems can yield side benefits.

• Making progress on core problems in one’s field can lead to connections with other fields.